## I B.Tech Supplementary Examinations, January 2014 MATHEMATICAL METHODS

( Common to Electrical \& Electronics Engineering, Mechanical Engineering,
Electronics \& Communication Engineering, Computer Science \&
Engineering, Electronics \& Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Control Engineering, Mechatronics, Computer Science \& Systems Engineering, Electronics \&
Telematics, Electronics \& Computer Engineering, Production Engineering,
Instrumentation \& Control Engineering and Automobile Engineering)
Time: 3 hours
Max Marks: 80

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Express the following system in matrix form and solve by Gauss elimination method.
$2 \mathrm{x}_{1}+\mathrm{x}_{2}+2 \mathrm{x}_{3}+\mathrm{x}_{4}=6 ; 6 \mathrm{x}_{1}-6 \mathrm{x}_{2}+6 \mathrm{x}_{3}+12 \mathrm{x}_{4}=36$,
$4 \mathrm{x}_{1}+3 \mathrm{x}_{2}+3 \mathrm{x}_{3}-3 \mathrm{x}_{4}=-1 ; 2 \mathrm{x}_{1}+2 \mathrm{x}_{2}-\mathrm{x}_{3}+\mathrm{x}_{4}=10$.
(b) Show that the system of equations $3 \mathrm{x}+3 \mathrm{y}+2 \mathrm{z}=1$; $\mathrm{x}+2 \mathrm{y}=4$;
$10 y+3 z=-2 ; 2 x-3 y-z=5$ is consistent and hence solve it.
2. A square matrix A is defined by $\mathrm{A}=\left[\begin{array}{ccc}-1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0\end{array}\right]$. Find the modal matrix P and the resulting diagonal matrix D of A .
3. Find the rank and index of the quadratic forms and reduce it to canonical form $3 x^{2}+5 y^{2}+6 z^{2}-2 x y+2 x z-2 y z$
4. (a) Given $u_{1}=22, u_{2}=30, u_{4}=82, u_{7}=106, u_{8}=206$, find $u_{6}$. Use Lagrange's interpolation formula.
(b) Find a real root of $\mathrm{x}^{3}-\mathrm{x}-2=0$.
5. (a) Find the best fitting straight line to the data:

| $\mathrm{x}:$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{y}:$ | 10 | 14 | 19 | 25 | 31 | 36 | 39 |

(b) Evaluate $\int_{0}^{4} e^{x} d x$ using trapezoidal and Simpson's rule. Also compare your result with the exact value of the integral.
6. Obtain $\mathrm{y}(0.6)$ and $\mathrm{y}(0.8)$ given $y^{\prime}=\mathrm{x}+\mathrm{y}, \mathrm{y}(0)=1$ with $\mathrm{h}=0.2$ by Adam's method.
7. (a) Expand $f(x)=x^{2}$ in $(-\pi, \pi)$ as a Fourier series and deduce the relations
i. $1+\frac{1}{2^{2}}+\frac{1}{3^{2}}+\frac{1}{4^{2}}+\ldots \ldots \ldots=\frac{\pi^{2}}{6}$.
ii. $1-\frac{1}{2^{2}}+\frac{1}{3^{2}}-\frac{1}{4^{2}}+\ldots \ldots \ldots=\frac{\pi^{2}}{12}$.
(b) Obtain the Fourier series expansion of $f(x)$ given that $f(x)=1-x$ in $-1<x<1$ and deduce the value of $\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\ldots \ldots \ldots$ [8+8]
8. (a) Find $\mathrm{Z}^{-1}\left(\frac{z^{2}-3 z}{(z+2)(z-5)}\right)$
(b) Solve $\mathrm{y}^{2} \mathrm{p}-\mathrm{xyq}=\mathrm{x}(\mathrm{z}-2 \mathrm{y})$.

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1. (a) Use matrix method to solve the equations $2 x-y+3 z=9, x+y+z=6, x-y+z=2$.
(b) Test for the consistency of the following system of equations and solve (if consistent) $\mathrm{x}+2 \mathrm{y}+\mathrm{z}=4,5 \mathrm{x}+8 \mathrm{y}+\mathrm{z}=14$. [8+8]
2. Using Cayley-Hamilton relation obtain the inverse of the matrix $\left[\begin{array}{lll}1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1\end{array}\right]$ [16]
3. Reduce the quadratic form $3 x^{2}-2 y^{2}-z^{2}+12 y z+8 z x-4 x y$ to canonical form by an orthogonal reduction and state the nature of the quadratic form.
4. (a) Solve $\mathrm{e}^{x}-3 \mathrm{x}=0$ by the method of iteration.
(b) Using Newton-Raphson method, find a positive root of $\mathrm{x}^{3}-\mathrm{x}-1=0$. $[8+8]$
5. (a) It is known that x , y are related by $\mathrm{y}=\frac{a}{x}+b x$ and the experimental values are given below:

| x: | 1 | 2 | 4 | 6 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| y: | 5.43 | 6.28 | 10.32 | 14.86 | 19.5 |

Obtain the best values of a and b.
(b) Find the first two derivatives of the function tabulated below at $x=0.6$

| x: | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| y: | 1.5836 | 1.7974 | 2.0442 | 2.3275 | 2.6511 |

6. Solve numerically using Euler's method and Taylor's method $y^{\prime}=\left(x^{3}+x y^{2}\right) / e^{x}$ given that $y(0)=1$. Find $y(0.1), y(0.2)$ and $y(0.3)$.
7. (a) Expand $f(x)= \begin{cases}1 ; & 0<x<\pi \\ 0 ; & \pi<x<2 \pi\end{cases}$
as a Fourier series.
(b) Obtain the Fourier series expansion of $\mathrm{f}(\mathrm{x})$ given that $f(x)=\left\{\begin{array}{ll}1 ; & 0<x<1 \\ 2 ; & 1<x<3\end{array}\right.$ and $\mathrm{f}(\mathrm{x})=3 / 2$ when $\mathrm{x}=0,1,3$ and $\mathrm{f}(\mathrm{x}+3)=\mathrm{f}(\mathrm{x})$ for all x.
8. (a) Solve $\mathrm{x}^{4} p^{2}+y^{2} z q=2 z^{2}$
(b) Find the inverse Z-transform of $\frac{\left(z^{2}-1\right) z}{\left(z^{2}+1\right)^{2}}$ using residues.

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1. (a) Find the rank of $\left(\begin{array}{ccccc}1 & 3 & 2 & 5 & 1 \\ 2 & 2 & -1 & 6 & 3 \\ 1 & 1 & 2 & 3 & -1 \\ 0 & 2 & 5 & 2 & -3\end{array}\right)$
(b) Solve completely the system of equations

$$
\begin{equation*}
4 x+2 y+z+3 u=0,6 x+3 y+4 z+7 u=0,2 x+y+u=0 \tag{8+8}
\end{equation*}
$$

2. (a) Determine the eigen values and eigen vectors of the matrix $A=\left[\begin{array}{ll}5 & 4 \\ 1 & 2\end{array}\right]$
(b) If $A=\left[\begin{array}{ll}2 & 0 \\ 0 & 1\end{array}\right]$, find $A^{100}$
3. Find the transformation that will transform $10 x^{2}+2 y^{2}+5 z^{2}+6 y z-10 z x-4 x y$ into a sum of square and find its reduced form.
4. (a) Find a positive root of $3 x-\sqrt{1+\sin x}=0$ by iteration method.
(b) If $y=(3 x+1)(3 x+4) \ldots \ldots \ldots \ldots(3 x+22)$, prove that
$\Delta^{4} \mathrm{y}=136080(3 \mathrm{x}+13)$
$(3 \mathrm{x}+16)(3 \mathrm{x}+19)(3 \mathrm{x}+22)$.
5. (a) Fit a parabola to the data given below

$$
\begin{array}{cccccc}
\mathrm{x}: & 1 & 2 & 3 & 4 & 5 \\
\mathrm{y}: & 10 & 12 & 8 & 10 & 14
\end{array}
$$

(b) For the table below:
find $f^{\prime}(1.76)$ and $f^{\prime}(1.72)$.

| $\mathrm{x}:$ | 1.72 | 1.73 | 1.74 | 1.75 | 1.76 | $[8+8]$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(\mathrm{x})$ | 0.17907 | 0.17728 | 0.17552 | 0.17377 | 0.17204 |  |

6. (a) Using Taylor series method obtain the values of y at $\mathrm{x}=0.2$ and $\mathrm{x}=0.4$ correct to 4 decimal places, if $y$ satifies the equation $\frac{d^{2} y}{d x^{2}}=x y$ given that $y^{\prime}=1$ and $\mathrm{y}=1$ when $\mathrm{x}=0$.
(b) Find y for $\mathrm{x}=0.2,0.4$ given $\frac{d y}{d x}=1+y^{2}, \mathrm{y}(0)=0$.
7. (a) Expand $f(x)=\left\{\begin{array}{l}x ; 0<x<\pi \\ 2 \pi-x ; \quad \pi<x<2 \pi\end{array}\right.$ as a Fourier series of periodicity $2 \pi$, and deduce the value of $\frac{1}{1^{2}}+\frac{1}{2^{2}}+\frac{1}{3^{2}}+\ldots \ldots . .=\frac{\pi^{2}}{6}$.
(b) Expand $\mathrm{f}(\mathrm{x})=-1$, in $(-2,0)$ and $\mathrm{f}(\mathrm{x})=1$, in $(0,2)$ as a Fourier series. $[8+8]$
8. (a) Solve the difference equation, using Z-transform $y(k+2)+2 y(k+1)+y(k)=u(k)$, where $\mathrm{y}(0)=0, \mathrm{y}(1)=0$ and $\mathrm{u}(\mathrm{k})=\mathrm{k}$ for $\mathrm{k}=0,1,2, \ldots \ldots$
(b) Solve $x^{2}(z-y) p+y^{2}(x-z) q=z^{2}(y-x)$ [8+8]

## Set No. 4

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Telematics, Electronics \& Computer Engineering, Production Engineering, Instrumentation \& Control Engineering and Automobile Engineering) Time: 3 hours

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Solve the system of the equations: $5 x+3 y+3 z=48 ; 2 x+6 y-3 z=18 ; 8 x-3 y+2 z=21$.
(b) Test for consistency and hence solve $\mathrm{x}+5 \mathrm{y}+7 \mathrm{z}=15,2 \mathrm{x}+3 \mathrm{y}+4 \mathrm{z}=11$, $x-2 y-3 z=-4,3 x+11 y+13 z=25$.
2. (a) Find the characteristic equation of the matrix $A=\left[\begin{array}{ccc}1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1\end{array}\right]$ Hence find $\mathrm{A}^{-1}$.
(b) Prove that $\sin ^{2} \mathrm{~A}+\cos ^{2} \mathrm{~A}=1$ where $\mathrm{A}=\left[\begin{array}{cc}1 & 2 \\ -1 & 4\end{array}\right]$
3. Reduce the quadratic form $3 x^{2}+5 y^{2}+3 z^{2}-2 y z+2 z x-2 x y$ to the canonical form and specify the matrix of transformation.
4. (a) Solve for a positive root of $\mathrm{x}^{3}-4 \mathrm{x}+1=0$ by Regula Falsi method.
(b) Represent the function $\mathrm{f}(\mathrm{x})=\mathrm{x}^{4}-12 \mathrm{x}^{3}+42 \mathrm{x}^{2}-30 \mathrm{x}+9$ and its successive differences in factorial notation in which the interval of differencing is one.

$$
[8+8]
$$

5. (a) Fit a curve $y=\mathrm{ae}^{b x}$ to the data by the method of least squares:

$$
\begin{array}{lccc}
\mathrm{x}: & 0 & 2 & 4 \\
\mathrm{y}: & 5.012 & 10 & 31.62
\end{array}
$$

(b) Compute $\int_{0}^{4} e^{x} d x$ by Simpson's one-third rule with 10 subdivisions. [8+8]
6. (a) Obtain $y(0.25)$ and $y(0.5)$ given $y^{\prime}=\frac{x^{2}}{1+y^{2}} y(0)=0$ by Picard's method.
(b) Apply Taylor's method to obtain the approximate value of y at $\mathrm{x}=0.2$ for $y^{\prime}=2 \mathrm{y}+3 \mathrm{e}^{x}, \mathrm{y}(0)=0$.
7. (a) Obtain the Fourier series expansion of $\mathrm{f}(\mathrm{x})$ given that $\mathrm{f}(\mathrm{x})=\mathrm{kx}(\pi-\mathrm{x})$ in $0<\mathrm{x}<2 \pi$ where k is a constant.
(b) Find the Fourier series of peridiocity 3 for $\mathrm{f}(\mathrm{x})=2 \mathrm{x}-\mathrm{x}^{2}$, in $0<\mathrm{x}<3$. $\quad[8+8]$
8. (a) Form the partial differential equation by eliminating $f\left(x^{2}+y^{2}, 2 x y\right)=0$.
(b) Solve the difference equation, using Z-transform $\mathrm{x}(\mathrm{k})-\mathrm{ax}(\mathrm{k}-1)=\mathrm{u}(\mathrm{n})$.

